

Glenn F. Engen
 National Bureau of Standards
 Boulder, CO 80302

Abstract

Although the six-port measurement technique is rapidly gaining the attention of the microwave community, the theoretical development, to date, yields but limited intuitive insight into how the technique actually works.

This paper presents an alternative introduction to the general subject which provides this insight.

The so-called "six-port" approach to the measurement of microwave parameters (e.g., power, complex impedance, etc.) provides an attractive alternative to existing automated measurement schemes because the requirement for frequency conversion has been eliminated.^{1,2,3,4,5,6,7}

While the theory which has been developed applies to a six-port junction of arbitrary parameters, only a limited amount of insight into the question of choosing the design goals for the six-port has been obtained. It is the purpose of this paper to present an alternative introduction to the six-port measurement technique which yields improved insight into the design question.

The measurement problem is illustrated in Fig. 1. At the terminal plane there are three independent parameters, some or all of which may be required in a given application. These may be conveniently expressed as the magnitude of the emerging wave amplitude ($|b|$) and the complex ratio (Γ_ℓ) between the amplitudes of the incoming wave (a) and the emerging wave (b).

Following the general arguments outlined in Ref. 3, one can write equations for the observed power meter readings: ($P_3 \dots P_6$)

$$P_3 = |Aa + Bb|^2 \quad (1)$$

$$P_4 = |Ca + Db|^2 \quad (2)$$

$$P_5 = |Ea + Fb|^2 \quad (3)$$

$$P_6 = |Ga + Hb|^2. \quad (4)$$

Here A \dots H are complex constants whose values are determined by the design of the six-port.

As a first step it is usually desirable to make the response of one power meter proportional to $|b|^2$; not only does this provide a direct determination of one of the measurands, but also a means of stabilizing the level at the measurement port. In order to provide continuity with the terminology in prior papers, the port chosen for this role is number 4. Referring to equation (2), the first design objective is that $C = 0$, and to the extent that this is realized, equation (2) becomes

$$P_4 = |D|^2 |b|^2. \quad (5)$$

In order to explicitly display the measurands of interest, equations (1), (3), and (4) may be written

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$$P_3 = |A|^2 |b|^2 |\Gamma_\ell - q_3|^2 \quad (6)$$

$$P_5 = |E|^2 |b|^2 |\Gamma_\ell - q_5|^2 \quad (7)$$

$$P_6 = |G|^2 |b|^2 |\Gamma_\ell - q_6|^2. \quad (8)$$

where $q_3 = -B/A$, $q_5 = -F/E$, and $q_6 = -H/G$.

It is possible to let $|b|$ and Γ_ℓ represent a point in three-dimensional space and to discuss the problem in terms of three-dimensional geometry. A more convenient approach, however, is to first eliminate $|b|^2$ from equations (6), (7), and (8) by means of equation (5). This leads to a problem in two dimensions. Although equation (5) is only an approximation, it will prove convenient to initially treat it as exact and then consider the general case.

Elimination of $|b|^2$ between equations (5) and (6), for example, leads to

$$|\Gamma_\ell - q_3|^2 = \left| \frac{D}{A} \right|^2 \cdot \frac{P_3}{P_4}. \quad (9)$$

Let Fig. 2 represent the Γ_ℓ plane. Ordinarily, the terminations to be measured are passive ($|\Gamma_\ell| \leq 1$) so that Γ_ℓ falls within the unit circle as shown. For reasons which will emerge, it is convenient to assume initially that q_3 lies outside this circle. Given the measurement results P_3 , P_4 , and assuming q_3 and $|D/A|^2$ are known, the locus of possible values for Γ_ℓ is a circle with center at q_3 and whose radius, $|\Gamma_\ell - q_3|$, may be determined from equation (9).

In the same way equations (5) and (7) may be combined, and the radius of another circle, which contains Γ_ℓ , with center at q_5 determined. The situation is now as shown in Fig. 3. Here Γ_ℓ is determined by the intersection of the two circles. Two circles, however, intersect in a pair of points. In this example the second point falls outside the unit circle, and one is able to choose between the two solutions on the basis $|\Gamma_\ell| \leq 1$.

Thus far, no use has been made of P_6 , and the system may be considered a five-port rather than six-port. Before introducing P_6 , some additional observations on the five-port mode are of interest. As already noted, the five-port mode leads to a pair of

values for Γ_ℓ . Provided, however, that the straight line between q_3 and q_5 does not intersect the unit circle, one is assured that one of these roots will fall outside of it and (assuming a passive termination) may be rejected on this basis.

By further inspection of Fig. 3, one notes, for the value of Γ_ℓ used in this example, that the angle at which the circles intersect is rather small; and it is easily recognized that the position of Γ_ℓ , in a direction perpendicular to the line between q_3 and q_5 , has a high sensitivity to errors in $|\Gamma_\ell - q_3|$ or $|\Gamma_\ell - q_5|$. In the parallel direction, the sensitivity is appreciably less. Over the range of possible choices for Γ_ℓ , and in particular if Γ_ℓ moves around the perimeter of the unit circle, one can expect a considerable variation in these sensitivities or expected errors in a practical measurement system. Although the five-port measurement concept is technically sound, the prime purpose for this discussion has been to prepare one to better appreciate the benefits of a six-port vs. five-port approach. Because these improvements are substantial, the future for the five-port appears limited.

To continue, q_6 is chosen as shown in Fig. 4 and $|\Gamma_\ell - q_6|$ is determined from equations (8) and (5). This provides a third circle upon which Γ_ℓ must lie and which (ideally) must pass through the intersection of the other two circles as shown in Fig. 4. In practice, because of measurement errors, the three circles will not intersect in a point, and some sort of statistical weighting is useful. Although it is not within the scope of this paper to consider this aspect in detail, it is intuitively obvious that this additional detector has substantially enhanced the accuracy with which Γ_ℓ may be determined. Moreover, the double root ambiguity has also been resolved; no longer is it required that the line connecting q_3 and q_5 lie outside the unit circle.

In the discussion thus far, it has been assumed that equation (5) was satisfied; at best this is only approximately true. Although the general solution for a non-zero value for "C" is not difficult, its presentation does not fall within the scope of this paper. Its general impact upon the graphical picture sketched above, however, is easy to state. As before, Γ_ℓ is determined by the intersection of two or more circles. These are no longer centered at q_3 , q_5 , or q_6 , although this may still be a fairly good approximation.

As noted in an earlier paragraph, and referring again to equation (2), the first design objective ordinarily is that $C = 0$. This leads to equation (5). Although nothing has been said, thus far, about the choice of $|D|$, $|A|$, $|E|$, and $|G|$, it is immediately evident from inspection of equations (5) ... (8) that these are scale factors, which for a given signal at

the output port, determine the power levels at the several power meters. Ordinarily, these parameters are chosen such that these levels are compatible with the power meter characteristics.

The major design question centers around the choice of q_3 , q_5 , and q_6 . One representative set of values is shown in Fig. 4. Although it is appropriate to ask if a better choice would be to place one of the q 's, say q_3 , at the center of the unit circle, it is easy to show, in the case of bolometric detectors at least, that this is not the case.

It thus appears, from symmetry considerations, that q_3 , q_5 , and q_6 should be located at the vertices of an equilateral triangle whose center is at the origin. This calls for $|q_6| = |q_5| = |q_3|$, while the arguments differ by $\pm 120^\circ$. Thus the only remaining choice is the value of $|q_3|$. Again, it is easy to show, with the help of examples, that an ill-conditioned situation results if $|q_3|$ is either too large or too small. In a practical situation it appears that the value should lie in the range 0.5-1.5.

References

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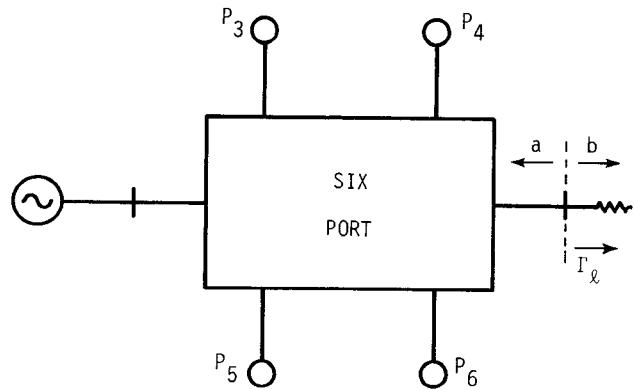


Fig. 1-Six-port for measuring complex microwave parameters.

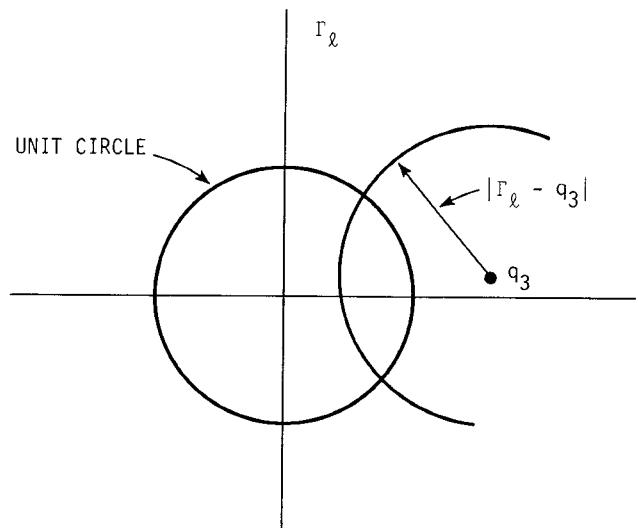


Fig. 2-Locus of possible values for Γ_ℓ determined by P_3 and P_4 .

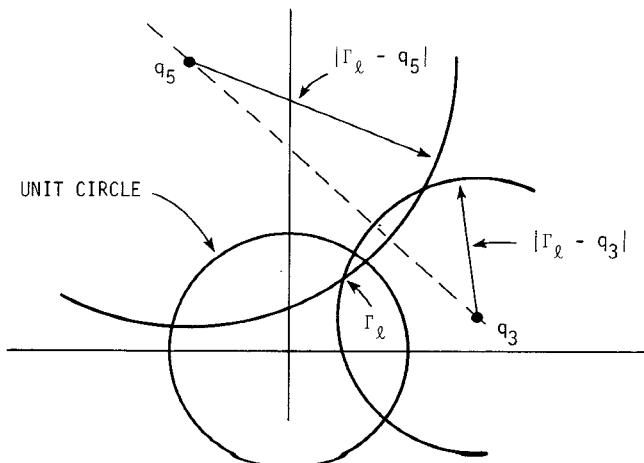


Fig. 3-Determination of Γ_ℓ from the intersection of two circles.

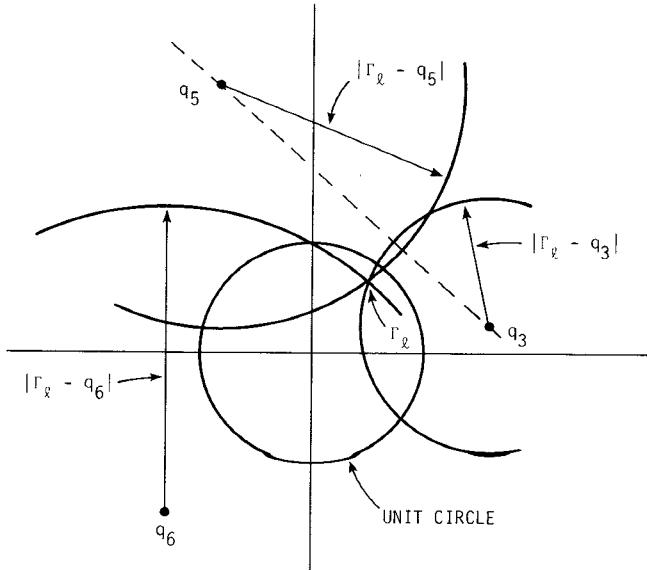


Fig. 4-An improved determination of Γ_ℓ from the intersection of three circles.